Equilibrium Timing of Price Announcements, With an Application to Internet Mortgage Data

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Abstract

This paper describes firms' decisions regarding the timing of price announcements. Consumers arrive at random, with arrival times following a given probability distribution. Arriving consumers observe price quotes posted at or before the time of their arrival and choose the best quote available. The fundamental trade-off that firms face is between the incentive for early announcement aimed at informing more consumers about the firm's price, and the incentive to wait for the rival firm to announce its price first. We characterize the unique symmetric mixed-strategy equilibrium of the timing game. Using the data from a price-comparison site for mortgages, we obtain the cumulative distribution of announcement times, fit it to the theoretical distribution, and then obtain a nonlinear least-squares estimate for the parameters of the underlying distribution of consumer arrival times.

Keywords: Non-Price Competition, Endogenous Timing of Moves, Game of Timing, Real-Time Price Adjustment.

JEL classification numbers: D43, L13, M3.

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"A great man always considers the timing before he acts."

Chinese proverb

1. Introduction

There exist plenty of studies on pricing decisions made by firms in alternative market environments.[1] However, the timing of price announcements is a much less researched area.[2] The previous literature on the subject mainly considers dynamic games in which players move sequentially. In pricing games, players usually prefer to be followers because moving after the opponents gives the information advantage.

Our model is closely related to the class of dynamic pricing games in which the order of moves is decided endogenously (Deneckere and Kovenock, 1992, for example). In contrast to most of these studies, we do not restrict the timing decision to a choice between two (or even a finite number of) periods. We allow the time variable to have a continuous support. Compare this to Ball and Cecchetti (1988), where firms decide to change their prices in odd or even periods. Another important difference from Ball and Cecchetti (1988) is that instead of looking at monopolistically competitive firms, we look at a homogeneous product oligopoly.[3] In tight oligopolies, firms realize their inter-dependence. They prefer to advertise prices promptly after their rivals to benefit from the information about the actual

[1] See, for example, Varian (1980), Sobel (1984), Lazear (1986), and Pashigian (1988).

[2] Timing games arise in a variety of settings, e.g., in the R&D competitions, patent races, price leadership models, entry and exit games.

[3] Ball and Cecchetti (1988) address the issue of the equilibrium timing of price changes in a large economy. They show that uniform staggering can arise as a stable Nash equilibrium in a model that endogenizes decisions of monopolistically competitive firms to change their prices. Staggering in firms' price setting is due to imperfect information about local and aggregate demand shocks.

rivals choice of price (which they can undercut). However, waiting is costly since a firm's price is not known to consumers until the firm posts the price.

Okun (1981) describe firms' incentives to wait till other firms set wages, and then respond promptly. According to Okun (1981), this "generates a tendency to spread the distribution of wage-adjustment dates around the calendar." We formally show, that, indeed, mixed strategy equilibrium is the only equilibrium that can arise when firms prefer to move just after their rivals, but waiting is costly due to the loss of prospective buyers.

A number of empirical studies compare price levels and dispersion in traditional and on-line markets. Price adjustment is a less explored topic.[4] Arbatskaya and Baye (2002) analyze the data from an intermediary, *Microsurf* , which posts the mortgage quotes of a number of lenders on the Internet. They find that online mortgage rates at *Microsurf* are not adjusting every time the T-Bill rate (a proxy for lenders' marginal cost of funds) is changing. While the T-Bill rate changed on more than 85 percent of days in the data, only about 16 percent of lenders' quotes represent a change in their posted mortgage rate. Mortgage rates are more rigid in concentrated markets. A closer look at the exact timing of rate changes is warranted.

Internet technology allows firms to observe the pricing decisions of other firms and to update their own quotes in real time. Our data, which came from the price comparison site, *Microsurf*, is rich in that it includes not only the information on mortgage rates and mortgage characteristics but also on the exact time the quote is submitted by the sellers. This allows for the analysis of timing games played by mortgage providers. We explicitly model the decisions of sellers regarding the

[4] For an empirical study on time-differentiated products see Borenstein and Netz (1999).

timing of price announcements.

More specifically, firms choose the best time to post their prices from a continuum of possible times.[5] It is showed that there is no pure strategy equilibrium in timing and the solution is sought in mixed strategies. Intuitively, firms try to make the timing of their moves unpredictable by randomizing with respect to time. We explicitly characterize the symmetric mixed strategy equilibrium and derive the conditions for its existence and uniqueness. Moreover, we estimate the parameters of the distribution of price announcement time with the data on Internet mortgage rates assuming that the distribution comes from a family of truncated Beta distributions. The estimated cumulative distribution fits closely the empirical one. Finally, we derive an underlying distribution of consumer arrival time, which gives rise to a Beta-distributed equilibrium timing strategy of firms. We show that consumer arrival time follows a well-defined cumulative distribution function that is uniquely determined.

The rest of the paper is organized as follows. In Section 2, we model the decision of a firm on when to announce its price, prove the nonexistence of a pure strategy equilibrium in this game, and derive a Pareto dominant symmetric mixed-strategy equilibrium, given a pre-specified distribution of the consumer arrival times. The model is estimated in Section 3 with the data on firms' mortgage rates posted by *Microsurf* on its web site. In Section 4, we estimate the parameters of the equilibrium mixed strategy using real time data and then derive the underlying distribution of consumer arrival times. Section 5 concludes. The Appendix shows how the model can be generalized to describe the competition between three or

[5]The pricing decision itself is trivial in the homogenous-product setting. The leader has an incentive to set its price equal to the consumers' willingness to pay and obtain monopoly profits till the follower draws all the consumers by undercutting the leader's price.

more firms.

2. The Timing Game

Consider a market in which two firms choose prices and the timing of price announcements. Each decides when to announce its price without knowing when the rival firm announces its price. At the same time, the price quote of the leader is observed by the follower. Consumers arriving at time t observe prices posted earlier in the day.

There are two types of consumers: patient consumers and impatient ones. Impatient consumers visit the site where firms post their prices only once. If no prices are available, they exit the market. If one price is available, they accept that price if it is below their reservation price. If multiple prices are available, they purchase at the lowest price. We assume that there is a unit measure of impatient consumers, and that their arrival times are described by a continuous distribution on $[0,1]$ with density $g(t)$ and corresponding cumulative distribution function, $G(t)$. Thus, $G(t)$ denotes the fraction of impatient consumers who show up before time $t \in [0,1]$, $G(1) = 1$ is the total measure of impatient consumers, and $G(0) = 0$.[6]

In addition to impatient consumers, there also exists a (possibly zero) measure $\mu \geq 0$ of consumers who are patient. They are willing to wait until both firms list their price on the site. Alternatively, one can interpret patient consumers as "last-minute shoppers" who are willing to wait until the last minute ($t = 1$) to check prices. Thus, the total measure of consumers is $1+\mu$. For simplicity, we model this

[6]It is easier to work with $G(x) = \Pr(X < x)$. Then $G(0) = 0$ and we assume that there is no atom at $t = 1$, $G(1) = 1$. When the distribution is continuous the exact definition of $G(\cdot)$ does not matter.

partition of consumers as exogenous, but one could easily endogenize consumer behavior by assuming that these two types of consumers have different discount rates, waiting costs, or more generally different opportunity costs. Without loss of generality, we assume that consumers have unit demand up to a reservation price $r \in (0, \infty)$ and that firms produce at zero cost. Firms must determine the timing of their price announcement as well as their announced price.

The fundamental trade-off that firms face is between the incentive for early announcement aimed at informing more consumers about the firm's price, and the incentive to wait for the rival firm to announce its price first.

The timing of decisions, as well as the information available at each decision point, is as follows. First, firms independently but simultaneously determine the time $t_i \in [0, 1]$ at which they will post their price $p_i \in (0, \infty)$. The firm that is the first to post its price cannot condition its price on that charged by the rival. In contrast, the second mover can observe the price that was posted by the rival. For simplicity, we assume that if the first and second mover charge the same price, consumers observing those prices will purchase from the firm whose price quote is the most recent. In this case, the optimal price by the second mover is that charged by the first mover. Note that if consumers used an alternative tie-breaking rule, the second mover would have an incentive to undercut the first-mover's price by a small amount, and thus $p_i(t_i) = r$ and $p_j(t_j) = r - \varepsilon$ for all $t_i < t_j$. For small ε, the differences in the first-mover and second-mover's prices would be negligible and might be ignored.[7]

[7] In this case, of course, there is no optimal price for the second mover if the strategy space is continouous (due to the usual open set problem, undercutting by $\varepsilon/2$ yields even higher profits). If the strategy space is discrete and the smallest unit of account is ε, then the second mover's optimal price is ε lower than the first mover's price.

This observation permits us to write the following "reduced form" for the timing game:

$$\Pi_i\left(t_i,t_j\right)=\begin{cases}(G(t_j)-G(t_i))r & \text{if} \quad t_i<t_j\\ \frac{1}{2}(\mu+1-G(t_i))r & \text{if} \quad t_i=t_j\\ (\mu+1-G(t_i))r & \text{if} \quad t_i>t_j\end{cases} \tag{1}$$

Intuitively, if firm i is the first to announce its price, then it only attracts those impatient consumers who arrive in the interval $[t_i, t_j]$. This is because impatient consumers who arrive before t_i never return and those who arrive at or after t_j will purchase from firm j. Being first precludes firm i from attracting patient consumers. Thus, the total demand for firm i is $(G(t_j) - G(t_i))$. Since the optimal price is the consumers' reservation price, the firm's profits are $(G(t_j) - G(t_i))\, r$.

Similarly, if firm i is the second to post its price, the firm will capture all μ of the patient consumers. In addition, it will attract all of the impatient consumers arriving after t_i. Since there are $(1 - G(t_i))$ of these latter consumers, total profits in this case are $(\mu+1-G(t_i))r$. In the event both firms announce the same price at the same time, consumers allocate themselves evenly among the firm, thus giving rise to the expression above for the case where $t_i = t_j$.

Since the pricing strategy for each firm is to announce the reservation price, in the analysis below we concentrate on the issue of timing in price announcements, normalizing the reservation price, r, to one. By the usual arguments, one can show that there does not exist a pure-strategy equilibrium whereby firms announce their prices at predetermined times. To avoid the problem of the non-existence of the best element in an open set in a continuous space, consider time as a discrete space and define δ as the smallest lapse of time, $\delta \in (0, 1)$.

Lemma 1. For sufficiently small δ, there does not exist a pure strategy equilibrium in the timing game.

Proof. First, suppose in a pure strategy equilibrium one of the firms chooses to announce its price immediately ($t_i = 0$). Then, the other firm's best response is to announce its price at $t_j = \delta$. This is not an equilibrium however. Firm i would rather set $t_i = 2\delta$ since $1 - G(2\delta) + \mu > G(\delta)$ for sufficiently small δ (since $G(2\delta) > G(\delta)$, it suffices to require $G(2\delta) < 1 + \mu$).

Next, consider $t_k \in (0; G^{-1}((1+\mu)/2))$, $k = 1, 2$. Since for $t_j \geq G^{-1}((1+\mu)/2)$ the best-response of firm i is $t_i = 0$, we only have to consider $t_j < G^{-1}((1+\mu)/2)$. In this case, the best-responses are $t_i = t_j + \delta$ and $t_j = t_i + \delta$. There does not exist a mutual best-response in this case as well. ■

Suppose each firm determines the timing of its price announcement based on an atomless distribution function, $F(t)$, with the support on the interval $[a, b] \subseteq [0, 1]$. The expected profit of firm i moving at time $t_i = t$ is equal to

$$\pi_i = \Pr(t > t_j)(\mu + 1 - G(t)) + \Pr(t < t_j)(E(G(t_j)|t_j > t) - G(t))$$

since the probability of two firms announcing a price at the same moment of time is zero for atomless equilibrium strategies. Given that firm $j \neq i$ is following mixed strategy $F(\cdot)$, the expression for firm i's profits can be re-written as

$$\pi_i = F(t)(\mu + 1 - G(t)) + (1 - F(t))(\int_t^b \frac{G(z)}{(1 - F(t))} dF(z) - G(t))$$

Simplifying yields

$$\pi_i = F(t)(1 + \mu) + \int_t^b G(z) f dz - G(t) = k \qquad (2)$$

Lemma 2. For any $\mu \leq (e-1)^{-1}$ and $k \in [\mu, (1+\mu)/e]$ there exists a symmetric mixed strategy equilibrium in the timing gamencements with equilibrium payoff k. The equilibrium mixed strategy has an atomless distribution with density

$$f(t) = \frac{g\,(t)}{1+\mu-G(t)} \tag{3}$$

and distribution function

$$F(t) = \ln k + 1 - \ln(1+\mu-G(t)) = \ln\left(\frac{ke}{1+\mu-G(t)}\right) \tag{4}$$

both defined on the support $\left[G^{-1}\left((1+\mu)-ke\right), G^{-1}\left((1+\mu)-k\right)\right]$.

Proof. Differentiate the profit function with respect to time to obtain $f(t)(1+\mu)-G(t)f(t)-g\,(t)=0$ and derive equation (3) for density $f(t)$. The distribution function is $F(t)=\int_a^t f(x)dx = c-\ln(1+\mu-G(t))$. If the density has as a support a time interval $[a,b] \subseteq [0,1]$ then from the boundary conditions it follows that

$$F(a) = 0 = c - \ln(1+\mu-G(a)) \tag{5a}$$

and

$$F(b) = 1 = c - \ln(1+\mu-G(b)) \tag{5b}$$

For a pair $(F(t), F(t))$ to constitute a symmetric mixed strategy Nash equilibrium (MSNE), we must ensure that

$$\pi_i(t) = F(t)(1+\mu) + \int_t^b G(z)f dz - G\,(t) = k$$

almost everywhere on the support of the MSNE, $t \in [a,b]$, and $\pi_i(t) \leq k$ outside the support, for $t \notin [a,b]$. In particular, at $t=b$ the profits equal to k, $\pi_i(b) = (1+\mu) - G\,(b) = k$. It follows that

$$b = G^{-1}\left((1+\mu) - k\right)$$

From (5b) $c = \ln(1+\mu - G(b)) + 1 = \ln k + 1$. Substituting the expression for c into (5a), we find

$$a = G^{-1}\left((1+\mu) - ke\right)$$

The support of the density of price announcement times $[a, b]$ belongs to a time interval $[0, 1]$, when $(1+\mu) - ke \geq 0$ and $(1+\mu) - k \leq 1$. That is, when $k \in [\mu, (1+\mu)/e]$. The condition $\mu \leq (e-1)^{-1}$ insures that $\mu \leq (1+\mu)/e$. ■

Proposition 1. Suppose $\mu \leq (e-1)^{-1}$. There exists a unique Pareto dominant symmetric mixed strategy equilibrium in the timing game defined by an atomless distribution with density as in equation (3) and distribution function

$$F(t) = \ln\left(\frac{1+\mu}{1+\mu - G(t)}\right) \tag{6}$$

on the support $[0, G^{-1}\left((1+\mu)(1-e^{-1})\right)]$. The profits in the equilibrium, $k^* = (1+\mu)/e$, increase in the number of patient consumers, μ.

Proof. The highest level of profits sustainable in a symmetric mixed strategy equilibrium is $k^* = (1+\mu)/e$. It is supported by the mixed strategy $F(t) = \ln\left((1+\mu)/(1+\mu - G(t))\right)$ defined on $[0, G^{-1}\left((1+\mu)(1-e^{-1})\right)]$. The density and distribution function outlined in equations (3) and (4) constitute a MSNE for $k^* \leq (1+\mu)/e$, that is whenever $\mu \leq (e-1)^{-1}$. The upper bound on profits, k^*, is increasing in the number of patient consumers, μ. ■

Note that a monopoly would earn $1+\mu$. Thus, even in the highest-profit outcome duopoly industry, the duopoly profits of $2(1+\mu)/e$ are lower than the monopoly profits by more than 26%. Catering to patient customers only and moving at $t = 1$ while the rival is randomizing according to $F(t)$, results in profits μ which are strictly lower than the profits obtained in any of the MSNE for $\mu > 0$.

Equation (6) gives the Pareto dominant equilibrium cumulative distribution of announcement times for any given cumulative distribution of consumer arrival times. From the observed cumulative distribution of announcement times we can derive the underlying distribution of consumer arrival times as follows:

$$G(t) = (1 + \mu)\left(1 - e^{1-F(t)}\right). \tag{7}$$

From (7), we obtain the density of the consumer arrival times,

$$g(t) = (1 + \mu)e^{-F(t)} f(t) \tag{8}$$

The next sections introduce the data and estimate $G(t)$ using a nonlinear least-squares procedure.

3. Data

The data, collected between April 30 and July 22 of 1998, includes 9777 daily observations on rates 92 different lenders charged for 30-year fixed mortgages with zero points[8] in all states in the United States and in the District of Columbia.[9] Interestingly, not all lenders posted rates in all states, nor did all lenders update their rates daily. Since our analysis focuses on incentives for lenders to change rates, and to avoid potential biases stemming from the inclusion of duplicate data

[8] A point, equal to one percent of the loan amount, is paid by a borrower at mortgage closing.

[9] The data was gathered at the *Microsurf*'s Internet site (*http://www.microsurf.com*) between 10 and 11 P.M. daily, except for May 15, May 16, and July 6.

from the same lender, only rates actually posted on the date of data collection are included in the sample.[10]

Figure 1 presents the histogram for the timing of price announcements for the entire sample.

[**Figure 1 HERE**]

Figure 2 depicts timing of rate changes by online lenders in all states between May 4, 1998 and May 7, 1998. The timing of price announcements is dispersed across time, and varies across states.

[**Figure 2 HERE**]

For example, on Tuesday of May 5, 1998, six lenders posted their current quotes in California: *Bankers Referral Group* at 12:49 p.m., *American Mortgage Association* at 3:36 p.m., *Loans4Less* at 3:59 p.m., *Interloan* at 4:34 p.m., *Avis Mortgage, Inc.* at 6:17 p.m., and *Hanover Mortgage Company* at 9:14 p.m.

4. Estimation

In this section we estimate the parameters of the equilibrium mixed strategy $F(t)$ using the data on mortgage rates posted in the real time on the Internet by *Microsurf*.[11] We first assume that the distribution of consumer arrival times

[10] Including outdated rates would have increased our sample size. As a practical matter, however, relatively few quotes were out of date. This likely stems from the following factors. First, we collected data at the end of each day. Second, lenders with the most up-to-date rates receive the benefit of being placed on top of the list. Finally, *Microsurf* warns consumers against relying on mortgage quotes that were not posted on the day of their search.

[11] *Microsurf* collects the information on the firms' advertised mortgage rates and provides it to buyers and sellers in all the states and in the District of Columbia.

is uniform, derive the symmetric equilibrium mixed strategy for this case, and estimate the parameters of this distribution. Then, we reverse the analysis: we consider a more flexible form of the equilibrium mixed strategy (beta distribution), fit it to data, and derive the underlying distribution of consumer arrival times.

4.1. Uniform distribution

Suppose consumers arrive at times distributed according to a uniform distribution, $t \sim U[0,1]$; the distribution and density functions are $G(t) = t$ and $g(t) = 1$, respectively. It follows from proposition (1) that a unique Pareto dominant symmetric mixed strategy equilibrium in the timing of price announcements is defined by an atomless distribution with density $f(t) = (1+\mu-t)^{-1}$ and distribution function $F(t) = \ln((1+\mu)/(1+\mu-t))$, on the time interval $[0,(1+\mu)(1-e^{-1})]$. Nonlinear least squares estimation yields the estimate for μ, $\widehat{\mu} = 0.5105032$. Hence, $\widehat{f}(t) = (11.5105 - t)^{-1}$ and $\widehat{F}(t) = \ln((1-t/1.5105)^{-1})$ on $t \in [0,(1+\widehat{\mu})(1-e^{-1})] = [0, 0.95482]$.

Since the empirical distribution does not fit closely the estimated time distribution, an alternative and more flexible form of the equilibrium time distribution should be considered.

4.2. Beta Distribution

Consider a distribution of price announcement times that follows a beta distribution truncated in the upper tail at $t = b$

$$f(t) = \frac{t^{\alpha-1}(1-t)^{\beta-1}}{\int_0^b x^{\alpha-1}(1-x)^{\beta-1}\,dx}$$

$$F(t) = \frac{\int_0^t x^{\alpha-1}(1-x)^{\beta-1}\,dx}{\int_0^b x^{\alpha-1}(1-x)^{\beta-1}\,dx}$$

for $t \in [0, b]$

We estimate the parameters of the distribution function $F(t)$, assuming it belongs to the family of truncated beta distributions. Parameters α, β and b are estimated by minimizing an integral of squared differences between the theoretical and the empirical cumulative distributions, $\widehat{F}(t)$ and $F(t)$, i.e., by solving the optimization problem: $\min\limits_{\alpha,\beta,b} \int_0^b \left(\widehat{F}(t;\alpha,\beta,b) - F(t)\right)^2 dt$.

Nonlinear least-squares procedure is used to estimate the coefficients α and β. For the mortgage rate data set, the estimates of α and β are $\alpha \approx 6.52$ and $\beta \approx 4.27$. Figure 3 depicts the estimated and the empirical distribution functions of price announcement times.

[**Figure 3 HERE**]

We infer what the distribution of the consumer arrival times must have been to generate the observed time distribution.

5. Conclusion

This paper is primarily devoted to questions of timing in price announcements. We argue that time is a major economic variable in itself. *When* to change a price is as important a question as, *what* the optimal price is?

In the model of this paper, each of the firms posts a price quote once during a period of time normalized to one. Consumers check the list of quotes and buy from a firm offering the lowest price, provided it is below the reservation price. The consumers are of two types, the patient and the impatient. The patient consumers wait till all the firms post their prices while the impatient buy only at the time of their arrival. Regardless of the time of firm's price announcement, the price the firm advertises is equal to the consumer willingness to pay, r. Thus, we can concentrate on how the firms decide on the timing of their moves. The firms face the following trade-off. When a firm moves, the firm observes all the previously announced prices, if any. Moving later, the firm is more likely to possess information about the rivals' actions. On the other hand, moving later the firm loses some potential impatient customers.

We prove that there does not exist a pure strategy equilibrium in the pricing game and characterize the equilibrium in mixed strategies. We show that there exists a well-defined distribution of consumer arrival times such that the theoretical distribution is a beta distribution. The empirical distribution of time fits well the beta-distributed equilibrium mixed strategy.

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6. Appendix A (Extension to $n > 2$ firms, $\mu = 0$)

Suppose that firms are randomizing according to some symmetric mixed strategy $F(\cdot)$. When there are n firms, the expected profit of firm i announcing its price at t, while all other firms follow the symmetric mixed strategy $t_j \sim F(\cdot)$, is the sum of the payoffs realized for each of the possible time ranking of the firm. With k firms advertising after firm i and $n-k-1$ advertising before it, the firm obtains the profits proportional to the measure of consumers who arrive between time it advertises, t, and the time of the earliest arrival of the successive firms, $E(G(t_{(1,k)})) - G(t)$. Here $t_{(1,k)}$ denotes the first order statistic (the minimum) of k independently distributed variables $t_j \sim F(\cdot)$.

$$\pi_i(t) = \sum_{k=0}^{n-1} \Pr(k \text{ firms move after firm } i) \cdot \Big(E(G(t_{(1,k)}) \mid t_{(1,k)} \geq t) - G(t)\Big)$$

Claim 1.

$$\pi_i(t) = \sum_{k=1}^{n-1} \binom{k}{n-1} F^{n-k-1}(t) \cdot \int_t^1 G(z)(1-F(z))^{k-1} f(z)dz + F^{n-1}(t) - G(t)$$

Proof.

The expected profit of firm i announcing its price at t can be re-written as

$$\pi_i(t) = \sum_{k=1}^{n-1} \binom{k}{n-1} F^{n-k-1}(t)(1-F(t))^k \cdot E(G(t_{(1,k)}) \mid t_{(1,k)} \geq t) + F^{n-1}(t) - G(t)$$

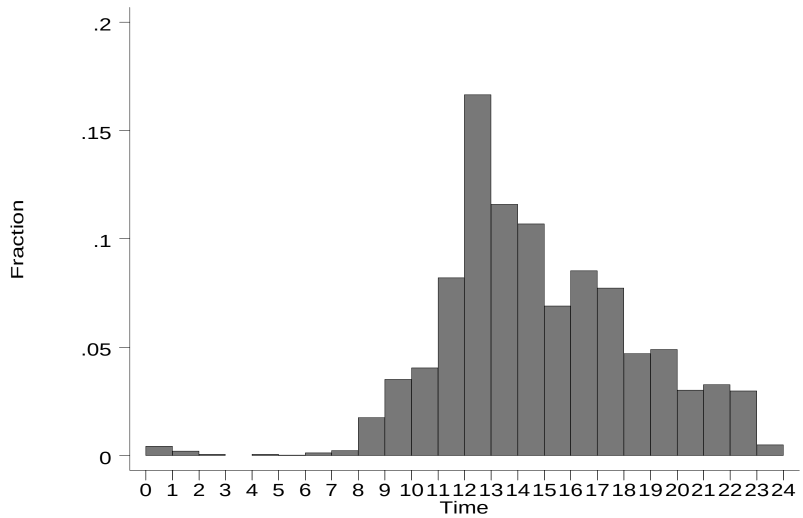
Next, we need to calculate, $E(G(t_{(1,k)}) \mid t_{(1,k)} \geq t)$, the expected value of the c.d.f. of the earliest arrival (among k firms) given that it is after time t.

The minimum of k independently distributed variables t_j (c.d.f. $F(\cdot)$ and p.d.f. $f(\cdot)$) has a density function $f_{\min} = k(1-F)^{k-1}f$. Then, $E(G(t_{(1,k)})) = k\int_0^1 G(z)(1-F(z))^{k-1}f(z)dz$. The expected value of $G(\min(t_j))$, given that $\min(t_j) \geq t$ is

$$E(G(t_{(1,k)}) \mid t_{(1,k)} \geq t) = \frac{\int_t^1 G(z)(1-F(z))^{k-1}f(z)dz}{(1-F_{\min}(t))} = \frac{\int_t^1 G(z)(1-F(z))^{k-1}f(z)dz}{(1-F(t))^k}$$

■

Figure 1. The Histogram for the Times of Mortgage Rate Announcements



Note: The time interval starts at 0, which corresponds to midnight.

Figure 2. The Timing of Mortgage Rate Announcements Across States Between May 4, 1998 and May 7, 1998

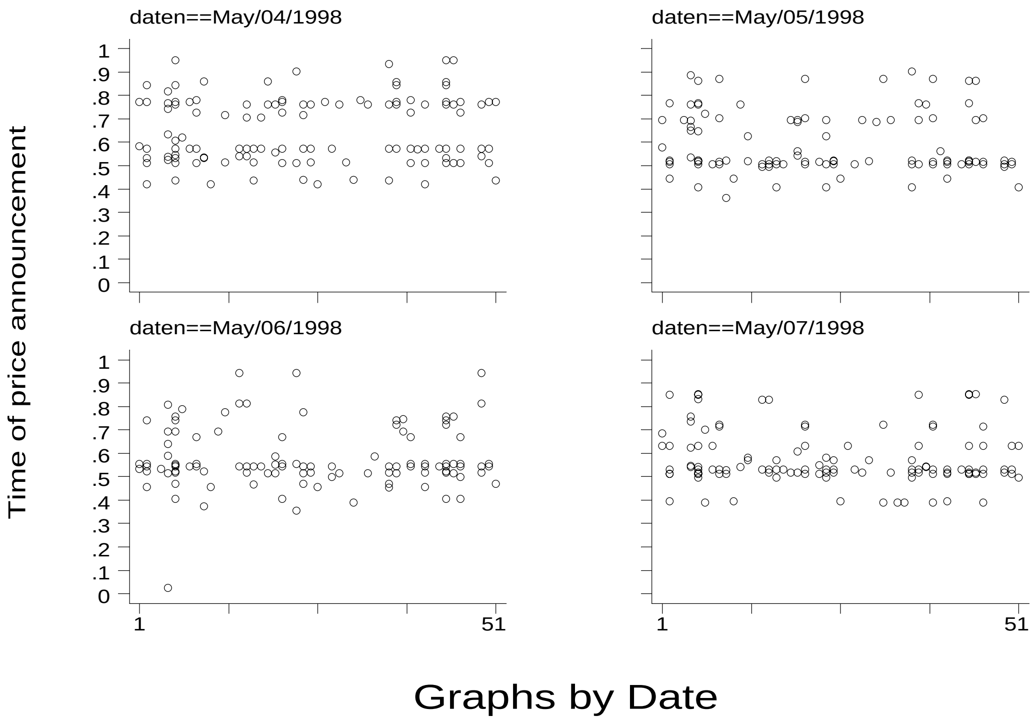


Figure 3. The Empirical and the Estimated Theoretical Cumulative Distribution of Timing